A NEW METHOD TO ESTIMATE THE MAGNITUDE RATIO

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This article introduces Ueki's method for calculating the magnitude ratio r. Ueki (1987) showed that the maximum likelihood estimate of the magnitude ratio r is given by the formula :

$$r = 1 + \frac{1}{m - M(m)} \tag{1}$$

where M(m) is the mean magnitude of meteors brighter than magnitude m. The numbers are corrected for the observers propability function.
Ueki's method converges more quickly than the conventional way of finding r :

$$r = \frac{m+1}{N(m)} \tag{2}$$

where N(m) is the number of meteors. Both M(m) and N(m) have to be corrected for the missed fraction of meteors, given by the observers probability function. Assuming a 'standard' probability function as found by Kresáková [2] a numerical example of the procedure and an application for some japaneese η-Aquarid observations are given.

Numerical example

Let us suppose somebody sees 39 meteors with the magnitude distribution n(m) as given in table 1.
The absolute numbers of meteors N(m) that appeared in the field of view are found by deviding the observed numbers of meteors n(m) by this probability function P(m) [2].
The mean magnitude of meteors brighter than magnitude m is given as M(m) By inserting this result in Ueki's equation (1) we get an estimation for r.

An application

Of course the result for r depends on the probability function that is assumed. A reasonable result is obtained if the r-values are the same as those found from many reported meteors. From the numerical example above we have an r increasing with m which therefor inplies that the probability function is not correct.
It is now assumed that the shape of the probability function is correct, but the function is shifted along m due to a non perfect limiting magnitude. From this assumption we may check the limiting magnitude estimate of the observer. It is well know, that a few observers report unreasonably high or low limiting magnitudes [3]. Magnitude distribution of meteors may help us to get a more reliable limiting magnitude as well as sporadic meteor rates.

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Table 2 gives the shifted probability function for several limiting magnitudes. The intermediate probabilities are estimated by using the simple interpolation in which a curve of second degree (a parabola) is applied. Table 3 gives the result of Ueki's method for observations of η-Aquarids by one experienced and two inexperienced Japaneese observers. The first series shows that :

- If we take the limiting magnitude as +6.8, the r-values decrease with increasing magnitude.
- If we take the limiting magnitude as +5.8, the r-values increase with increasing magnitude.
- If we asume the r-value to be constant between +2 and +4, we might suppose the limiting magnitude was 6.0±0.2 and the r-value is 2.3±0.1.

The reported limiting magnitude was 6.5±0.3 which does not differ significantly. Series (2) in table 3 show the result of an inexperienced observer. He mentioned the limiting magnitude to be +5.0, but this value gives an unreasonable behaviour of r(m). The r-value indicates, that the limiting magnitude was more like 6.8 and r about 2.0. It is easy to evaluate the reliability of an observer's magnitude estimates.
Series (3) are for an inexperienced observer. The r-values fluctuate heavily for every limiting magnitude assumption. This may also be due to the low number of meteors observed, but the example shows that one should be carefull when using such data.
Table 4 compares the results for r-values from different methods [4]. The agreement is bad but improve if the true mean magnitude of sporadic meteors i.e. that observed by an observer that has the 'standard' probability function, is somewhat less than assumed than assumed in methods (2) and (3) in table 3 (See table caption).
Here we use the 'standard' probability, but it is obvious, that the probability differes from observer to observer as well as from condition to condition. It is necessary to confirm whether the 'standard' probability function may be applied or not when we use this method. •

References

[1] Ueki, K.: *"NMS Halley Project Review 1" (1987)4 (In Japanese)*

[2] Kresáková, M.: *Contr. Astron. Obs. Skalnaté Pleso 3(1966) pg. 75.*

[3] Spalding, G.: *J. Brit. Astron. Assoc. 98(1987) pg. 26.*

[4] Jenniskens, P.: *DMS Visueel Handboek. DMS 1988.*

m	-3	-2	-1	0	+1	+2	+3	+4	+5
n(m)	1	1	0	2	3	5	11	13	3
P(m)	0.87	0.73	0.57	0.48	0.420	0.343	0.232	0.064	0.008
N(m)	1.2	1.4	0.0	4.2	7.1	15	47	203	375
M(m)	-3.0	-2.5	-2.5	-0.9	+0.1	+1.1	+2.3	+3.5	+4.5
r	∞	3.2	1.7	2.1	2.1	*2.1*	*2.4*	*3.1*	2.6

Table 1: *Numbers of observed meteors n(m), Probability functions P(m), corrected numbers of meteors N(m) and cumulative numbers M(m)*

L_m	6.5	6.3	6.1	5.9	5.7	5.5
-4	0.950	0.938	0.925	0.909	0.890	0.870
-3	0.870	0.843	0.816	0.788	0.759	0.730
-2	0.730	0.692	0.657	0.625	0.596	0.570
-1	0.570	0.549	0.530	0.512	0.495	0.480
0	0.480	0.469	0.458	0.446	0.433	0.420
+1	0.420	0.407	0.393	0.377	0.361	0.343
+2	0.343	0.325	0.305	0.283	0.258	0.232
+3	0.232	0.189	0.151	0.117	0.088	0.064
+4	0.064	0.048	0.035	0.024	0.015	0.008
+5	0.008	0.005	0.003	0.002	0.001	-

Table 2: *The standard probability function shifted proportional to the difference (limiting magnitude - 6.5).*

m	n(m)	6.8	*6.0*	5.8	n(m)	7.3	*6.8*	6.2	n(m)	*6.5*	6.0	5.0
-1	1	-	-	-	2	-	-	-	1	3.1	3.1	3.3
-2	0	2.0	2.0	2.0	0	2.0	2.0	2.0	0	1.7	1.7	1.7
-1	4	4.5	4.6	4.6	1	1.8	1.9	1.9	2	2.1	2.1	2.1
0	20	5.8	5.7	5.6	1	1.7	1.7	1.7	3	2.1	2.1	2.1
+1	40	3.4	3.4	3.5	4	*2.1*	*2.1*	*2.1*	5	*2.0*	*2.0*	*2.1*
+2	40	2.3	*2.3*	*2.4*	3	*1.8*	*1.8*	*1.8*	11	*2.2*	*2.3*	*2.9*
+3	53	*2.0*	*2.3*	*2.5*	10	*2.1*	*2.2*	*2.5*	13	*2.1*	*2.4*	*4.1*
+4	29	*1.8*	*2.4*	*2.7*	3	1.7	1.8	2.1	3	1.7	2.0	3.1
+5	4	1.6	2.0	2.3	3	1.8	2.2	3.6	0	-	-	-

Table 3: *Application of Ueki's method of estimating r for a magnitude distribution of η-Aquarids by one experienced and two unexperienced observers.*

											magnitude ratio		
m	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	(1)	(2)	(3)
η-Aqr	0	1	0	4	20	40	40	53	29	4	2.35	3.00	2.77
sporadics	1	1	0	2	8	14	22	32	24	1	2.53	-	(3.4 def.)

Table 4: *A comparisson of different methods to determine r.*
(1): Ueki's Method, proposed in this article.
(2): From the normalized mean magnitude ($\overline{x}_s$=3.25).
(3): From the ratio of stream to sporadic numbers per magnitude.