ASTRONOMY & ASTROPHYSICS
SUPPLEMENT SERIES

JULY **1995**, PAGE 173

Astron. Astrophys. Suppl. Ser. **112,** 173-178 (1995)

A new positional astrometric method for all-sky cameras

J. Borovička, P. Spurný and J. Keclíková

Astronomical Institute, 251 65 Ondřejov Observatory, Czech Republic

Received December 9, 1994; accepted January 27, 1995

Abstract. — All-sky photographs, used in meteor astronomy, need a special transformation formulae for converting the plate coordinates to celestial coordinates. An improved form of these formulae is presented. It includes a new radial projection function and the influence of the inclination of the plate from the plane perpendicular to the optical axis. The method then enables to achieve the precision near the theoretical limit simultaneously for the entire visible hemisphere. About 15 reference stars spread on the whole sky are needed. The meaning of the individual reduction constants is explained in detail. Practical hints for using the method and numerical examples are given.

Key words: astrometry — meteors — methods: data analysis

1. Introduction

1.1. *All-sky cameras*

All-sky photographic cameras are used in meteor astronomy for imaging the entire visible hemisphere in one frame. Their advantage is not only that all (sufficiently bright) meteors appearing above the horizon can be recorded but also that even a very long meteor is always completely present on only one plate which enables a consistent positional astrometric reduction.

In this paper we are dealing mainly with the cameras equipped with the fish-eye lenses Zeiss Distagon 3.5/30 mm. Such cameras are used in part of the European Fireball Network. The diameter of the image of the sky on the plate is 80 mm. The cameras are used either in the fixed mode or in the guided mode. Examples of photographs obtained by them are presented in Fig. 1. Fixed cameras are used for the computation of meteor atmospheric trajectories, guided cameras enable to determine the time of meteor passage. The reference objects for positional reduction are the beginnings and ends of star trails and the star images, respectively.

All-sky cameras need not be used only for meteor observations. In general, they may be useful for detection of all sufficiently bright objects appearing on random positions on the sky. Our photographs have been already used for the search for optical flashes from γ-ray bursters (e.g. Hudec et al. 1987). Bright novae may be another possibility.

Due to highly non-linear imaging of all-sky cameras, the classical astrometric methods cannot be used. The most appropriate formulae for the global transformation of the plate rectangular coordinates x, y to the celestial coordinates have been searched in a series of papers (Ceplecha 1987; Spurný 1987; Borovička 1992). Here we present, as we hope, the final version of these formulae. They enable the determination of the celestial coordinates on the whole hemisphere with the precision near the theoretical limit. The formulae may be applied not only to the Zeiss fish-eye lenses but to all types of all-sky optical systems, e.g. to concave mirrors cameras.

Send offprint requests to: J. Borovička, Internet address: borovic@asu.cas.cz

1.2. *The present astrometric method*

The most advanced method published up to now is that of Borovička (1992). It may be summarized as follows:

The projection is assumed to be axially symmetrical. We define the angular distance from the center of projection (i.e. from the optical axis), u, and the "azimuth of projection", b. The u ranges from 0 to 90°, the b from 0 to 360°. If the center of projection were identical with zenith, u would be identical with the zenith distance, z, and b would correspond to the astronomical azimuth, a. In reality, the camera is never oriented precisely toward the zenith, the center of projection lies at a (small) zenith distance ε, and azimuth E. The astronomical coordinates can be obtained from the projection coordinates by the relations

$$\cos z = \cos u \cos \varepsilon - \sin u \sin \varepsilon \cos b\,, \qquad (1)$$

$$\sin(a - E) = \sin b \sin u / \sin z \qquad (2)$$

(the zero point of b is defined so that for the zenith is $b = 180°$). The origin of the plate coordinates is set near the (a priori unknown) center of projection. The x axis is oriented toward the south and the y axis toward the

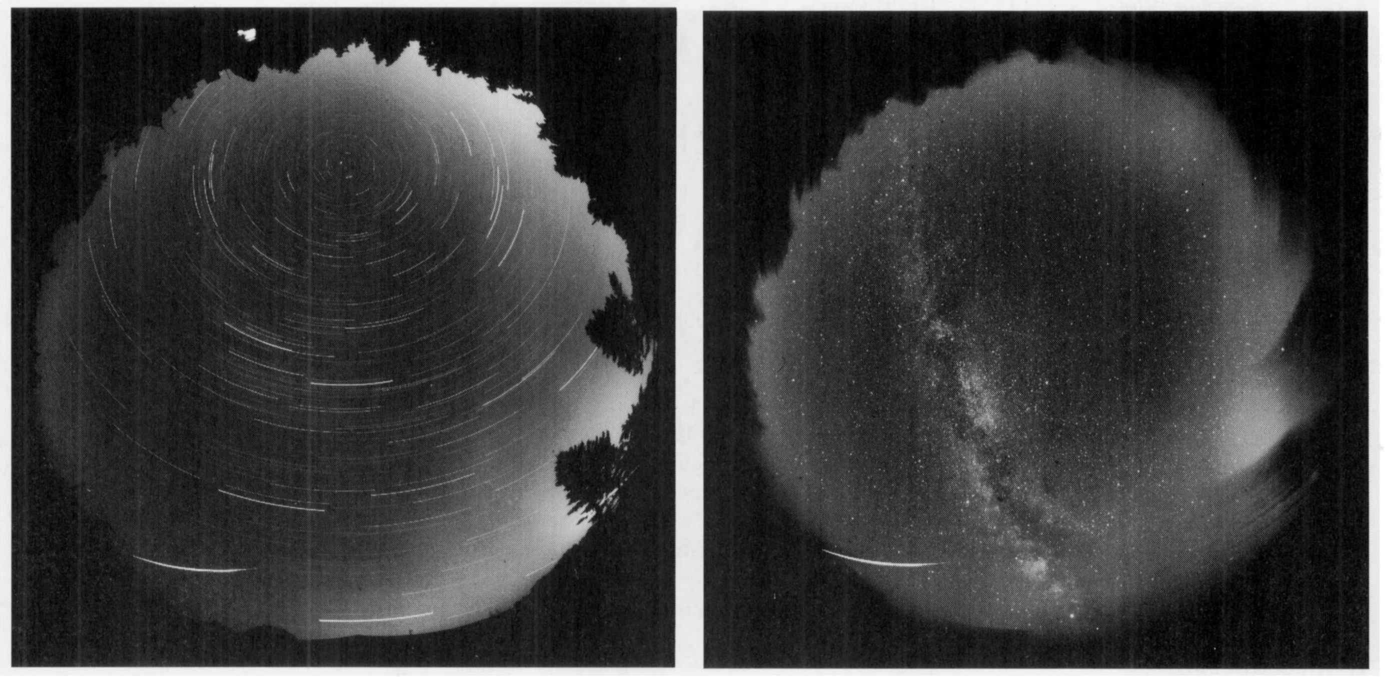

Fig. 1. The photographs from the fixed (on the left) and the guided all-sky camera at station No. 20. Both were obtained with the Zeiss Distagon lens. The exposures were 1.5 hour long. Limiting magnitudes for stars are 5 in the fixed image (in the equatorial region) and 10 in the guided image. The fireball *Valeč* (see Sect. 3) is present in the southeast

west. The relations between the plate coordinates and the projection coordinates are supposed to be

$$u = Vr + S(\mathrm{e}^{Dr} - 1)\,, \qquad (3)$$

$$b = a_0 - E + \arctan\left(\frac{y - y_0}{x - x_0}\right), \qquad (4)$$

where

$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2}. \qquad (5)$$

Here x_0, y_0 are the coordinates of the center of projection in the x, y system and a_0 is the angle between the x axis and the direction to the south. The constants V, S, D describe the plate scale as the function of the radial distance from the center, r.

The above five equations represent the transformation formulae from the measured coordinates x, y to the azimuth and zenith distance. They contain eight unknown transformation constants: $a_0, x_0, y_0, V, S, D, \varepsilon, E$.

2. Description of the new method

2.1. *Improving the formulae*

The above equations had been used for the computation of trajectories of meteors photographed in scope of the European Fireball Network. Sometimes, we noticed that the convergence of the solutions from different stations is still not satisfactory. This was particularly true for the stations where the meteor was observed in large zenith distances. We therefore suspected that the radial scale Eq. (3) is not good enough for large r and performed thorough tests.

2.1.1. The radial scale formula

Any theoretical or factory function for the radial projection of the objective is not known. The Eq. (3) is empirical. In fact, there is never enough reference objects (the beginnings of star trails) at large zenith distances ($z > 80°$) in the stationary meteor photographs. Fitting the formula to stars and using it for a meteor which is close to the horizon means an extrapolation. To obtain a good idea on the behavior of the radial function far from the optical axis, we had to use photographs from the guided cameras for the test.

A photograph from the guided camera shows the sky at the position in the middle of the exposure time (more precisely, at the time when the hour angle of the camera was zero). However, the parts of the sky which were close to the horizon at that time were much higher at some other time during the exposure. This enabled that stars at 90° from the optical axis could be exposed, especially in the western and eastern part (and even more than 90°, the field of view is actually by few degrees larger than 180°!).

We have used four well exposed guided plates from two different cameras for the test. 64–126 stars were measured on each plate with the Zeiss Ascorecord measuring device.

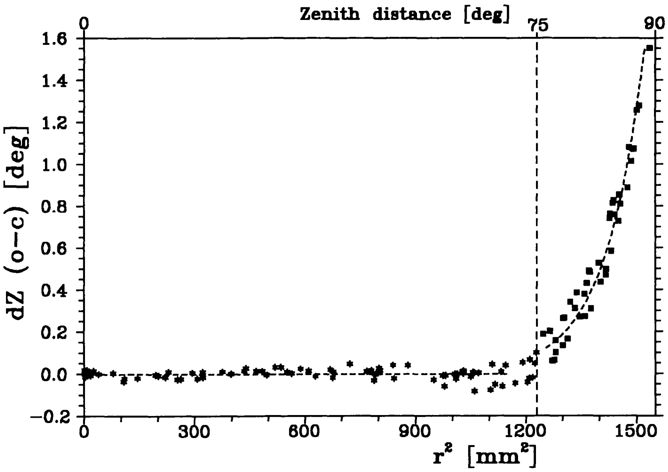

Fig. 2. The residuals in zenith distances as a function of r^2 for the plate 20A–2305 using the old method (Borovička 1992) for stars with $z < 75°$ only. The residuals for stars with $z > 75°$ have been fitted with an exponential function

We chose numerous stars near the horizon but stars over the whole hemisphere were included.

We then reduced the plates with the Borovička (1992) method and displayed the residuals. It was immediately found that the method is unable to fit the positions of the stars on the whole plate. The results for all four plates were very similar. In Fig. 3a the residuals in zenith distance are given as a function of the zenith distance for the plate 20A-2305. A large systematic trend, reaching a good fraction of a degree at almost all zenith distances, is present.

Another test was therefore performed. The method was run only for stars with zenith distances less than 75°. The residuals were, however, displayed for all stars (see Fig. 2). For stars with $z > 75°$ this represents an extrapolation, similarly as it was sometimes done for meteors. As expected, the stars with $z < 75°$ were well fitted, but the residuals for stars with $z > 75°$ grew exponentially and exceeded one degree and a half at $z = 90°$. The residuals are displayed in Fig. 2 as a function of the square of the plate radial distance, r^2. An exponential function was fitted to the residuals of stars with $z > 75°$. It can be seen that the fit represents the data well. This was the reason we tried to change the Eq. (3) to

$$u = Vr + S(e^{Dr} - 1) + P(e^{Qr^2} - 1)\,, \qquad (6)$$

i.e. to add the second exponential term with two new reduction constants to be determined, P and Q. With this new equation, we obtained the residuals displayed in Fig. 3b. The systematic trend in zenith distances has been completely removed.

2.1.2. The radial distance formula

The last term in Eq. (6) eliminated the effect in zenith distances which motivated this study. However, we discovered another, quite unexpected, though much smaller, systematic effect. It is displayed in Fig. 4a. The residuals in azimuth are a function of the azimuth. After having made a few unsuccessful attempts to remove this effect, we have found that the effect is not caused by a failure of the lens axial symmetry, but by the different scale in different directions. The effect could be removed assuming that the radial distance, r, must be computed according

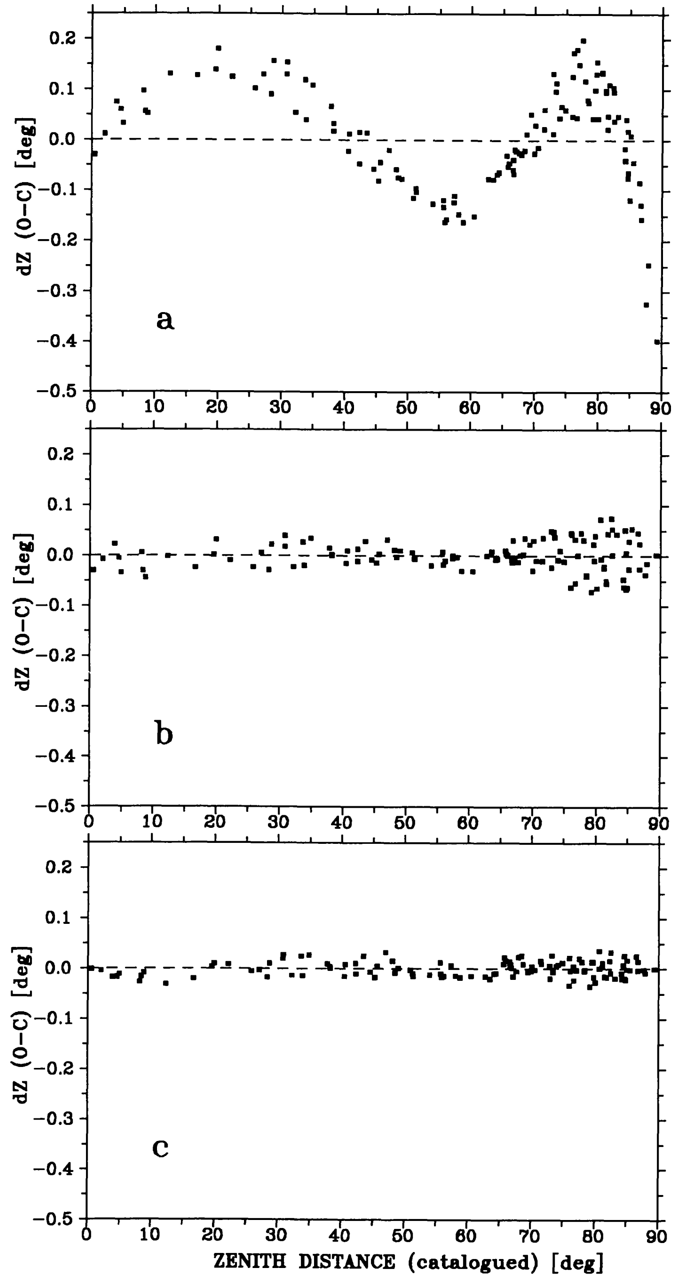

Fig. 3. The residuals in zenith distances for the plate 20A-2305 using three different methods: a) the old method (Borovička 1992); b) after introducing Eq. (6); c) the complete new method

to formula

$$r = r_{\rm old}[1 + A \sin(\alpha - F)], \qquad (7)$$

where $r_{\rm old}$ is given by Eq. (5) and α is defined as

$$\alpha = a_0 + \arctan\left(\frac{y - y_0}{x - x_0}\right). \qquad (8)$$

A and F are two new reduction constants, the amplitude and phase of the scale variation, respectively. It can be easily shown that the effect is caused by an inclination of the plate from the plane perpendicular to the optical axis. In that case the projection is elliptical with $r = p/(1 + e\cos\alpha)$, where p is the parameter and e is the eccentricity of the ellipse. In the limit $e \ll 1$ we obtain $r = p(1 - e\cos\alpha)$, i.e. a harmonic function in the form of (7). The constant A is proportional to the eccentricity, i.e. to the magnitude of the inclination and F is a phase which expresses the direction of the inclination. The removal of the systematic trend in azimuths after using Eq. (7) is demonstrated in Fig. 4b. Also the residuals in zenith distance became smaller, as it can be seen in Fig. 3c. The value of A in this case was 0.0012.

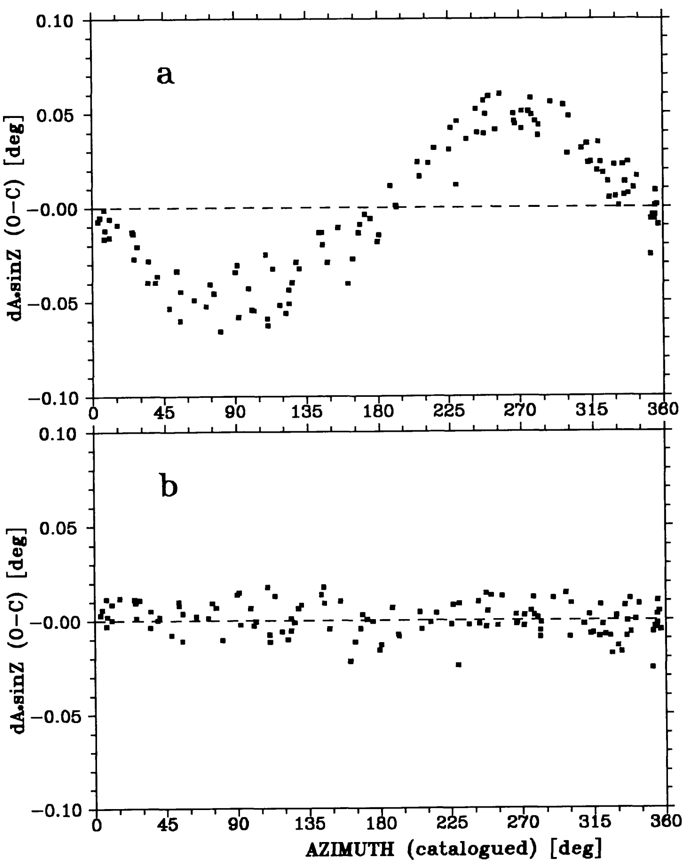

Fig. 4. The residuals in azimuths for the plate 20A-2305 using two different methods: a) with symmetrical radial distance function; b) the complete new method

2.2. *The summary of the new method*

2.2.1. The formulae

The transformation of the plate coordinates x, y to the celestial coordinates a, z is done by means of five equations. The equation for r can be rewritten as

$$r = C\left[\sqrt{(x - x_0)^2 + (y - y_0)^2} + \right. \qquad (9)$$
$$\left. + A(y - y_0)\cos(F - a_0) - A(x - x_0)\sin(F - a_0)\right],$$

where we introduced the global scale factor C (see below). The other four equations are

$$u = Vr + S(e^{Dr} - 1) + P(e^{Qr^2} - 1) \qquad (6)$$

$$b = a_0 - E + \arctan\left(\frac{y - y_0}{x - x_0}\right) \qquad (4)$$

$$\cos z = \cos u \cos\varepsilon - \sin u \sin\varepsilon \cos b \qquad (1)$$

$$\sin(a - E) = \sin b \sin u / \sin z \qquad (2)$$

The equations contain 13 (12 independent) reduction constants which must be determined or assumed for each plate. The unknown constants are computed by the least squares method minimizing the residuals in zenith distances $|z - z_{\rm cat}|$ and azimuths $|a - a_{\rm cat}| \sin z_{\rm cat}$ simultaneously. The index 'cat' refers to the catalogued values. As the equations are non-linear, they are linearized and the least squares method is used in successive iterative steps. The procedure is described in more detail for the old formulae in Borovička (1992).

2.2.2. Reduction constants

The reduction constants can be divided into four groups according to their physical meaning:

1. a_0, x_0, y_0 – the plate constants; they define the rotation and shift of the rectangular measuring coordinate system on the plate.
2. A, F, C – the camera constants; they define the position of the plate relative to the optical axis. A and F express the inclination of the plate and C expresses the shift of the plate along the axis. C is equal to unity when the center of the plate lies at a standard distance from the lens (the nominal focal length). Otherwise it differs a little from unity.
3. V, S, D, P, Q – the lens constants; they define the projection of the lens in dependence on the distance from the optical axis. The influence of astronomical refraction is also included.
4. ε, E – the station constants; they define the deviation of the camera optical axis from the zenith. An apparent deviation may be also caused by an incorrect timing.

The lens constants and the constant C are not independent and cannot be computed simultaneously. This can be seen, for example, from the fact that both V and C define

Table 1. The lens constants from four plates with standard deviations

plate	N_*	$u_{\max}$	σ	V	S	D	$P \cdot 10^6$	Q
20A-1803	64	90°6	0°0150	0.031656	0.00677	0.0953	2.20	0.00638
				±41	33	12	77	20
20A-2305	123	89°2	0°0137	0.031624	0.00716	0.0938	5.85	0.00579
				29	26	9	1.58	16
4A-128	124	89°3	0°0127	0.031458	0.00687	0.0942	2.63	0.00620
				25	26	8	69	15
4A-536	85	90°7	0°0136	0.031475	0.00666	0.0951	1.98	0.00636
				29	23	9	48	14

the scale at the center of projection in the same way. Either C is set to unity and the lens constants are computed or the lens constant are fixed and C is being determined.

The catalogued horizontal coordinates of reference stars are computed from the equatorial coordinates using some time. In case of fixed cameras this is the time of the beginning or the end of the exposure, in case of guided cameras the time when the hour angle was zero (the observer notices the hour angle at the beginning and the end of exposure). If the time is incorrect from some reason, an apparent increase of ε is the result. In case of guided cameras this has no consequences, because the final result are the equatorial coordinates of the measured object and the error is eliminated in the backward transformation. On the other hand, for meteors on stationary plates any error in timing is fatal. The resulting coordinates are then shifted in hour angle just by the value of the timing error.

As the apparent positions of reference stars are used as the catalogue values, the computed lens constants include the influence of astronomical refraction. We were unable to separate the effect empirically. Nevertheless, this is not necessary because in astronomical applications the object of interest is influenced by refraction as well.

The fish-eye objectives we use exhibit a chromatic aberation in radial direction far from the optical axis. The lens constants are therefore dependent on wavelength. The effect is not particularly large. Nevertheless, we recommend not to use extremely blue reference stars. If the object (meteor) contains bright blue emissions, they should be ignored in the positional measurement.

2.2.3. Numerical examples

The full method represents computation of 12 unknown reduction constants. The test cases showed us that the method converges well. However, large number of reference stars is needed. In particular, the reliable determination of P and Q requires stars at very large zenith distances which are seldom available. Sometimes even only few stars are present in the whole frame (images influenced by bad weather, strong moonlight, etc.). We have therefore developed a whole family of restricted reduction procedures in which some constants are kept fixed. The detailed description is beyond the scope of this paper. Usually, one of the two almost equivalent procedures are used: Either only P and Q are being kept at their standard values or all five lens constants are fixed and C is allowed to differ from unity. About 15 reference stars well spread in the whole frame are sufficient to obtain reliable results.

The lens constants as resulted from the full reduction of the four testing plates are given in Table 1. All distances are measured in millimeters and angles in radians. So, the dimension of V is rad mm^{-1} etc. The first two plates were from station No. 20, *Ondřejov*, the rest two from station No. 4, *Churáňov*. Also given is the number of reference stars used, N_*, the angular distance from the optical axis u for the farthest star, and the resulting standard deviation of one star position, σ.

The values of the lens constants are surprisingly consistent for all plates, except for P, Q in 20A-2305. The difference in V reflects the difference in plate scales at the two stations. We have defined the results for plate 20A-1803 as the standard lens constants. When they were used as fixed for the plate 20A-2305, the standard deviation increased only slightly (from 0°0137 to 0°0143), the difference in P, Q proved therefore insignificant. The standard deviations do not exceed 0°015. This corresponds to about 6 μm on the plate, a value quite comparable to the measuring errors.

3. Using the new method

The new method has been used on the four testing plates and on many regular plates with meteors. In most cases the standard deviation σ of about 0°015 was achieved. However, this standard deviations express only the internal precision among the set of reference stars. In fact, the method can be best tested on meteors. Meteors represent ideal independent testing objects. When the same meteor is photographed from three or more stations, the individual directions to the meteor, as measured along the meteor path on different stations, must all intersect near one line in the space – the real atmospheric trajectory of the meteor. If this is not the case the coordinate system on at least one station was adjusted incorrectly.

As an example of the improvement achieved with the new method, we present here the fireball EN 030884, *Valeč* (Ceplecha & Spurný 1987). This bright meteor was photographed on six Czech stations of the network. As the end point of the luminous trajectory lay below 20 km of height, the fireball was rather close to the horizon on more distant stations and the advantages of the new method became clearly visible.

In Fig. 5, the deviations of the individual directions (lines of sight), as measured on different stations, from the average meteor trajectory are displayed. The average trajectory has been computed by the straight least squares method (Borovička 1990).

In Fig. 5a the old astrometric method was used. The more distant stations Nos. 20, 4, and 11 are completely wrong at the end of the trajectory, where the meteor was at larger zenith distances than 75°. The deviation increases with increasing zenith distance and was clearly caused by the inadequacy of Eq. (3). The effect is not visible at another distant station No. 14, because the meteor was projected perpendicularly to the horizon here.

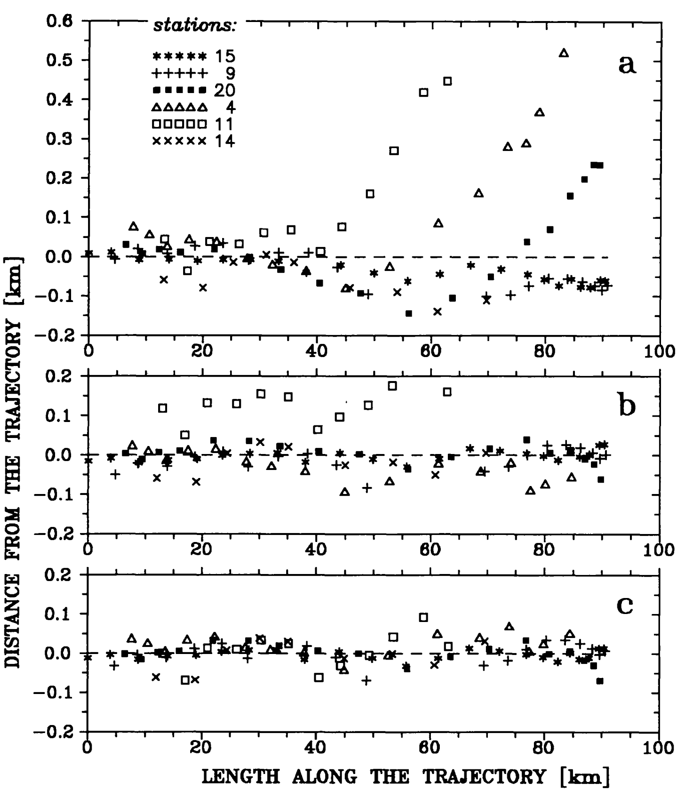

Fig. 5. The solutions for fireball EN 030884. Deviations of lines of sight from the average trajectory are displayed. Different astrometric methods were used: a) the old method (Borovička 1992); b) using the new radial scale formula; c) using also the new radial distance formula

After introducing Eq. (6), we obtained the deviations displayed in Fig. 5b. The zenith distance effect has been removed. However, small systematic trends still persist, especially for stations 11 and 4. The complete use of the new method removed also these trends, as it can be seen in Fig. 5c. The deviations are now almost random; of course, the more distant stations show larger scatter. The solution was obtained with the same standard values of the lens constants for all stations. This demonstrates the homogeneous quality of the optics.

It should be noted that the differences between the average solutions of the meteor trajectory with the old method (Fig. 5a) and the new one (Fig. 5c) are not fatal. The locations of the beginning and the end point of the meteor in space differ by 0.1 km and difference in radiants is $0\overset{\circ}{.}1$. Distant stations got automatically smaller weight in our trajectory computation and thanks to the close stations 15 and 9 the trajectory could be reasonably determined even with the old method in this case. However, some meteors photographed only from large distances could not be computed with the old method at all.

4. Conclusions

In addition to the previous method, the new method presented here introduces four new reduction constants, P, Q, A, F. The number of constants increased to 12. However, all constants are well justified. The necessity of the term containing P and Q in the radial scale formula was demonstrated in Fig. 3. The effect of plate inclination, which was ignored in previous methods, is well detectable, as demonstrated in Fig. 4. So, A and F are also needed. On the other hand, it was found that the five lens constants can be fixed for an objective of the given type and replaced by the constant C.

The presented method enables to achieve, at least for the tested type of objective, the precision near the theoretical limit given by the precision of measurement, simultaneously for the entire visible hemisphere. In this sense, it cannot be surpassed by any method used only locally. The only requirement is about 15 reference stars spread on the whole sky. The FORTRAN 77 subroutine with our method is available from authors on request.

Acknowledgements. This work was partly supported by the grant project No. 303406 from the Academy of Sciences of the Czech Republic.

References

Borovička J. 1990, Bull. Astron. Inst. Czech. 41, 391
Borovička J. 1992, Publ. Astron. Inst. Czech. Acad. Sci. 79, 19
Ceplecha Z. 1987, Bull. Astron. Inst. Czech. 38, 222
Ceplecha Z., Spurný P. 1987, Handbook for MAP (Middle Atmosphere Project) 25, 321
Hudec R., Borovička J., Wenzel W. et al. 1987, A&A 175, 71
Spurný P. 1987, eds. Z. Ceplecha, P. Pecina, Interplanetary Matter, Proceedings of the 10th E.R.A.M., Publ. Astron. Inst. Czech. Acad. Sci. 67, 225